Finding $A(s)$ from $|A(j\omega)|^2$

Consider $A(s) = a_n s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0$ with a_i real.

Define even and odd parts:

$$Ev\{A(s)\} = \frac{[A(s) + A(-s)]}{2}$$

$$Od\{A(s)\} = \frac{[A(s) - A(-s)]}{2}$$

Then $A(s) = Ev\{A(s)\} + Od\{A(s)\}$ and $Ev\{A(s)\} = Ev\{A(-s)\} = a_0 + a_2 s^2 + ...$ the sum of even powers.

In general, for even $F(s) = F(-s)$, $F(j\omega) = F(-j\omega)$ must be real.

$Od\{A(s)\} = -Od\{A(-s)\} = a_1 s + a_3 s^3 + ...$ the sum of odd powers.

Let $s \to j\omega$. Then $Ev\{A(s)\} = a_0 - a_2\omega^2 + a_4\omega^4 + ...$ is real and

$Od\{A(s)\} = a_1 j\omega - a_3 j\omega^3 + a_5 j\omega^5 + ... = j(a_1\omega - a_3\omega^3 + a_5\omega^5 + ...)$ is imaginary.

$Od\{A(j\omega)\} = j\,\mathrm{Im}\{A(j\omega)\}$.

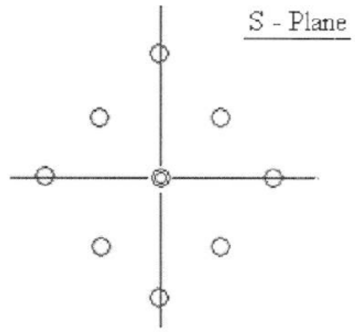
Hence

$$\begin{aligned}|A(j\omega)|^2 &= [\mathrm{Re}\{A(j\omega)\}]^2 + [\mathrm{Im}\{A(j\omega)\}]^2 \\ &= [\mathrm{Even}\{A(j\omega)\}_{s=j\omega}]^2 - [\mathrm{Odd}\{A(j\omega)\}_{s=j\omega}]^2 \\ &= A(s)A(-s)|_{s=j\omega}\end{aligned}$$

It is a polynomial in ω^2 only. ω^2, ω^4, ω^6

Given $|A(j\omega)|^2$, A(s) can be recovered as follows:

1. Replace ω^2 by $(S/j)^2 = -s^2$
   This Gives $A(s)A(-s) = 0$
   $K_o^2(s - s_1)(s - s_2) \ldots\ldots\ldots\ldots(s - s_{2n})$
   Since $A(s)A(-s)$ is a function of s^2 only, its zeros are symmetrical around the origin



2. Distribute the zero between A(s) & A(-s). If the zeros of A(s) must be in the LHP (i.e. If A(s) is a Hurwitz polynomial), Select only those, otherwise, there are multiple choices. (Complex conj. zeros stays together!)

3. The Constant factor may be +K or –K in A(s)

Example: Let $|A(j\omega)|^2 = \omega^4 + 1$
Then $A(s)A(-s) = s^4 + 1 = 0$
$(s + a + ja)(s + a - ja)(s - a + ja)(s - a - ja) = (s^2 + s/a + 1)(s^2 - s/a + 1)$
The Hurwitz factor is $\pm(s^2 + s/a + 1)$
Here, $a = 1/\sqrt{2}$ $\quad$ $s^2 + \sqrt{2}\,s + 1$